

P-1

[This question paper contains 7 printed pages]

Your Roll No. :

Sl. No. of Q. Paper : **1831** **GC-4**

Unique Paper Code : 32371208

Name of the Course : **B.Sc.(Hons.) Statistics**

Name of the Paper : Probability &
Probability
Distributions

Semester : II

Time : 3 Hours

Maximum Marks : 75

Instructions for Candidates :

- (a) Write your Roll No. on the top immediately on receipt of this question paper.
- (b) Attempt six questions in **all**.
- (c) Question No. 1 is compulsory.
From the remaining questions do any **five** questions by selecting at least **two** questions from **each** section.
- (d) Use of simple calculator is allowed.

1. (a) For what value of k ,

$$f(x) = \begin{cases} ke^{-2x} & ; x \geq 0, \\ 0 & ; x < 0, \end{cases}$$

is the probability density function of a random variable X .

P.T.O.

P2

(b) If X and Y are independent Cauchy variates with parameters (λ_1, μ_1) and (λ_2, μ_2) respectively then find the distribution of $(X + Y)$.

(c) If the characteristic function $\phi_X(t)$ of a continuous random variable X is given, then how will you find p.d.f $f(x)$?

(d) State the relationship between $M_X(t)$ and μ'_r .

If two variables X and Y are independent then what is $E(Y/X)$?

(f) Let X be a random variable with probability distribution :

$X = x$	-2	2	4
$P(X = x)$	1/8	1/2	3/8

Find $E(X)$ and $\text{Var}(X)$

(g) Let X and Y have the joint p.d.f.

$f(x, y) = c(2x + y); 0 \leq x \leq 1, 0 \leq y \leq 2$, then find c .

2

P3

(h) If $\text{Cov}(aX + bY, bX + aY) = ab \text{Var}(X + Y)$, then comment on the independence of X and Y .

(i) The probability distribution of a random variable X is :

$$P(X = x) = (1/2)^x; x = 1, 2, \dots$$

Then find mean and mode of X .

(j) If the p.d.f. of a random variable X is :

$$f(x) = c \exp[-(x^2 - 6x + 9)/24]; -\infty < x < \infty,$$

then find c , mean and variance of X .

Section - A

2. (a) Let X be a random variable with p.m.f. given by:

$X = x$	0	1	2	3	4	5	6	7
$P(X = x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

(i) find k ,

(ii) find $P(X < 6)$ and $P(0 < X < 5)$,

3

P.T.O.

P-4

(iii) if $P(X \leq a) > 1/2$, find the minimum value of a ,

(iv) determine the distribution function of X .

(b) For the distribution :

$$dF(x) = \begin{cases} y_0 \left(1 - \frac{1}{a} |x - b| \right) dx, & b - a < x < b + a \\ 0, & \text{otherwise} \end{cases}$$

calculate y_0 , mean and variance.

3. (a) Let X be a random variable taking non-negative integral values. If the moments of X are given by :

$$E(X^r) = 0.6; r = 1, 2, 3, \dots$$

then find m.g.f. of X . Also, show that $P(X = 0) = 0.4$, $P(X = 1) = 0.6$, $P(X \geq 2) = 0$.

(b) An urn contains balls numbered 1, 2, 3. First a ball is drawn from the urn and then a fair coin is tossed as many times, as the number shown on the ball drawn. Find the expected number of tails.

4

P-5

4. (a) Define the characteristics function of a random variable. Show that the characteristic function of sum of two independent random variables is equal to the product of their characteristic functions. Is the converse true, if not justify.

(b) If the joint p.m.f. of X and Y is given by :

$$p(x, y) = \frac{(x - y)^2}{7} \text{ for } x = 1, 2 \text{ and } y = 1, 2, 3.$$

find :

- the marginal distribution of Y ,
- the joint distribution of $U = X + Y$ and $V = X - Y$,
- the marginal distribution of U .

Section - B

5. (a) Find the m.g.f. of standard binomial variate $(X - np) / \sqrt{npq}$ and obtain its limiting form as $n \rightarrow \infty$. Also interpret the result.

5

P.T.O.

P-6

(b) Let X be the negative binomial variate with p.m.f.,

$$f(x) = \begin{cases} \binom{k+x-1}{x} q^x p^k & ; x = 0, 1, 2, \dots \\ 0 & ; \text{otherwise} \end{cases}$$

Show that the moment recurrence formula is :

$$\mu_{r+1} = q \left(\frac{d\mu_r}{dq} + \frac{rk}{p^2} \mu_{r-1} \right), r = 1, 2, \dots$$

Hence find variance of X .

6

6. (a) If X and Y are independent Poisson variates such that

$$V(X+Y) = 4 \text{ and } P(X=3 | X+Y=6) = 5/16,$$

then find $E(X)$ and $V(Y)$.

6

(b) If $X \sim N(\mu, \sigma^2)$, then find the p.d.f. of $Y = e^X$ and identify it. Also find the coefficient of variation of Y .

6